LECTURE SUMMARY 12.2

FRIDAY, JULY 29, 2016

Nonlinear System of Differential Equations

$$\begin{cases} x' = F(x,y) \\ y' = G(x,y) \end{cases} \tag{1}$$

1. How to find equilibrium points: let $x' = 0$ and $y' = 0$.

2. Linear approximation using Taylor expansion.(Not requiring.)

3. **Theorem** Suppose that F and G have continuous second derivatives. Then the stability of an equilibrium point (x_0, y_0) of (1) may be assessed by finding the **eigenvalues** of the Jacobian matrix

$$\begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{bmatrix}$$

evaluated at (x_0, y_0).
The stability may be described as follows.

Eigenvalues	Stability	Equilibrium points
Both positive	Unstable	Node (or possibly a spiral point for repeated eigenvalues)
Both negative	Asymptotically stable	Node (or possibly a spiral point for repeated eigenvalues)
Opposite signs	Unstable	Saddle point
$\alpha \pm \beta i$, $\alpha > 0$	Unstable	Spiral point
$\alpha \pm \beta i$, $\alpha < 0$	Asymptotically stable	Spiral point
$\pm \beta i$	Indeterminate	Center or Spiral point